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## Reply by Author to J. I. Erdos and H. Gold

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ERDOS and Gold, in the preceding comment, question the value of a simplified correlation of wake transition data[1]; however, the fact remains that the available ballistic range data are inconclusive with regard to model size dependence,[1,2–6] and small differences in body shape.[1,2,5] Thus, an involved correlation scheme based upon these data does not seem to be indicated at present. The behavior shown schematically in Fig. 1 of the foregoing comment is not demonstrated by the data (with the possible exception of a hint at the shape by two points of the sphere data[8,7] at $M_\infty \simeq 7.5$, i.e., region 7 on Fig. 2 of the preceding note) even though the trend may be implied by the initial results of stability investigations. Therefore, when using the data outside of the range of the experiments, one must be careful to avoid reading too much from the data; e.g., note the reversals in the effect of body size over the Reynolds number range for the cases shown in Fig. 3 of the preceding note (also see Fig. 1).

With regard to the comparison of correlations shown in Fig. 3 of the preceding comment, the author notes that, according to the curve-fit described in Ref. 9, the extrapolation of the "unified transition correlation" (Fig. 4 of Ref. 1 or Fig. 2 of Erdos and Gold's note) to $M_\infty = 22$ should indicate $(Re_{\bar{x}TR})^2_\infty\ (M_\infty/M_e) = 10^8$. For a 12° cone at $M_\infty = 22$, the quantity $M_e$ (as defined by the author in Ref. 1) is about 14.5; then $(Re_{\bar{x}TR})_\infty \simeq 4.3(10)^7$, and the comparison of the author's correlation and that of Ref. 2 appears as shown in Fig. 1. It is seen that, for the larger bodies, agreement to within a factor of 3 (which is within the experimental data scatter according to all available correlations) is obtained for altitudes of 200 kft and below.

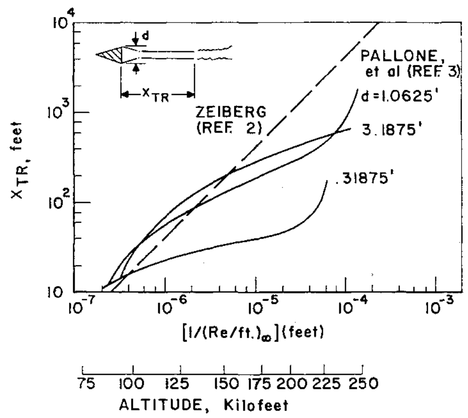


**Fig. 1 Wake transition predictions for 12° cone at 22,000 fps.**

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## Comment on "Derivation of Element Stiffness Matrices"

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REFERENCES 1 and 2 describe methods of structural analysis in which the deflections of an element are expressed in terms of a number $N = n + l$ basic displacement functions. The element is loaded and/or attached to other elements at its nodes, which have $n$ slopes and deflections. The difference between this and previous work is that $l$, the number of surplus shape functions, was previously assumed zero.

A comparison of Refs. 1 and 2 is of interest in that the solutions are independent, are expressed differently, and have different motivation. Pian remarks that by taking a large number $l$ of surplus undetermined coefficients $\alpha$, the equilibrium conditions are improved. Reference 1 suggests that the first few $\alpha$, say $\alpha_1 \ldots \alpha_r$, may represent the necessary rigid body motions; this guarantees exact equilibrium. Experience with large $l$ in a two-dimensional problem indicates that the difficulties of imposing conformity of slopes and deflections between elements increase with $l$.

In the $[G]$ of Ref. 2, the first $r$ rows and columns will be zero because the rigid body motions do not contribute to the strain energy. Some arithmetic may then be saved by using only the nonzero terms of $[G]$, say $[G_0]$, a $(n - r) \times (n - r)$ matrix. Because $\alpha_1 \ldots \alpha_r$ are of no subsequent interest, the first $r$ rows of $[B_a^{-1}]$ may be discarded leaving $[C_0]$, say, a $(n - r) \times l$ matrix. Then (8) of Ref. 2 becomes

$$\{\alpha^*\} = \begin{Bmatrix} \alpha^* \\ \alpha_b \end{Bmatrix} = \left[\begin{array}{c|c} \begin{matrix} C_0 \\ (n-r) \times n \end{matrix} & \begin{matrix} -C_0 B_b \\ (n-r) \times l \end{matrix} \\ \hline \begin{matrix} 0 \\ l \times n \end{matrix} & \begin{matrix} I \\ l \times l \end{matrix} \end{array}\right] \begin{Bmatrix} q \\ \alpha_b \end{Bmatrix} = [M^*] \begin{Bmatrix} q \\ \alpha_b \end{Bmatrix}$$

where the asterisk means that the first $r$ rows are missing.

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